

Implementation of Efficient algorithm for exact Hausdorff Distance

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ABSTRACT

The Hausdorff distance is a very important metric for various image applications in computer vision including image matching, moving-object detection, tracking and recognition, shape retrieval and content-based image analysis. However, no efficient algorithm has been reported that computes the exact Hausdorff distance in linear time for comparing two images. Very few methods have been proposed to compute the approximate Hausdorff distance with higher approximation error. In this paper, we propose a linear time algorithm for computing the approximated Hausdorff distance with lower approximation error. The proposed method is effective to reduce the processing time, while minimizing the error rate in content-based image processing and analysis. The Hausdorff distance is a measure of (dis-)similarity between two sets which is widely used in various applications. This has applications, for instance, in image processing. In this paper we propose a novel efficient algorithm for computing the exact Hausdorff distance. The proposed algorithm is tested against the HD algorithm of the widely used National Library of Medicine Insight Segmentation and Registration Toolkit (ITK) using magnetic resonance volumes with extremely large size. The proposed algorithm outperforms the ITK HD algorithm both in speed and memory required. The Hausdorff Distance differs from many other shape comparison methods in that no correspondence between the model and image is derived. The method is quite tolerant of small position errors such as those that occur with edge detectors and other feature extraction methods.

Keyword: Hausdorff Distance, Early Breaking, Random Sampling In Place Of Scanning, Excluding Intersection, Runtime Analysis.

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I. INTRODUCTION

The Hausdorff distance (HD) is a measure of dissimilarity between two point sets. The HD is an important metric that is commonly used in many domains like image processing and pattern matching as well as evaluating the quality of clustering. It is also used in medical applications like for detecting Breast Cancer. Most of the applied literature is devoted to the computation for sets consisting of a finite number of points. This has applications, for instance, in image processing. However, we would like to apply the Hausdorff distance to control and evaluate optimisation methods in level-set based shape optimisation. In this context, the involved sets are not finite point sets but

characterised by level-set or signed distance functions. For example we can calculate area or perimeter of any type of shape like polygon. As polygon is not having a fixed shape that's why there is no such a formula to calculate area or perimeter. But, by using this algorithm we can calculate the distance between two point sets.

There are various types of measures used to compare two point clouds. Overlap based measures, e.g. the Dice coefficient, consider an imaginary grid on the union of the two point sets and calculate the overlap (intersection) between the point sets with respect to the grid. Points are assigned to subsets depending on whether they are or are not in the intersection to build the confusion matrix.

Spatial distance based measures generally consider the pairwise distances between the compared point sets. Examples from this category are the average distance, i.e. the average of all pairwise distances, and the Mahalanobis distance, which compares estimates of the point sets as two hyper-ellipses. Both examples are also not sensitive to the positions of the individual points because in the case of the average distance, distances of far points are compensated by other near points and in the case of the Mahalanobis distance, estimating the point sets as hyper-ellipses means ignoring details of the positions of the points. The HD is a max-min distance, and hence has the advantage that it takes into consideration the spatial position of each individual point, which makes it capable of considering the spatial properties in the measurement, e.g. the boundary of an object.

The directed Hausdorff distance \tilde{H} between two point sets A and B is the maximum of distances between each point $x \in A$ to its nearest neighbour $y \in B$.

That is:

$$H(A, B) = \max_{x \in A} \{ \min_{y \in B} \{ \|x, y\| \} \} \dots \dots (1)$$

where $\|.,.\|$ is any norm e.g. the Euclidean distance function. Note that $\tilde{H}(A, B) \neq \tilde{H}(B, A)$ and thus the directed Hausdorff distance is not symmetric. The Hausdorff distance H is the maximum of the directed Hausdorff distances in both directions and thus it is symmetric.

H is given by:

$$H(A, B) = \max \{ \tilde{H}(A, B), \tilde{H}(B, A) \}$$

For a point set A, we define the point set size to be the number of elements in A. Images and image volumes are a special class of point sets, where the elements are pixels (or voxels for volumes) that are in pre-defined locations on a grid. For an image A, we define the point set size to be the number of pixels/voxels in A that are not in the background (non-zero pixels/voxels). Also we define the grid size to be the dimensions of the entire image including background (width x length x height). Note that the proposed algorithm is equivalently applicable on images and volumes, so we will not strictly differentiate between them. The same applies to pixel and voxel.

II. HAUSDORFF DISTANCE

The Hausdorff distance, or Hausdorff metric, also called Pompeiu–Hausdorff distance, measures how far two subsets of a metric space are from each other. It turns the set of non-empty compact subsets of a metric space into a metric space in its own right. It is named after Felix Hausdorff.

Informally, two sets are close in the Hausdorff distance if every point of either set is close to some point of the other set. The Hausdorff distance is the longest distance you can be forced to travel by an adversary who chooses a point in one of the two sets, from where you then must travel to the other set. In other words, it is the greatest of all the distances from a point in one set to the closest point in the other set.

HD algorithm should take into consideration how these two characteristics vary in relation to the following parameters:

Point set size:

For example, a brain MRI volume could reach a million voxels and that of a whole body could reach 10 million voxels. The runtime of the algorithm should stay reasonable when the set size increases extremely.

Grid size:

It is desirable that the complexity of the algorithm depends only on the point set size rather than the grid size. For example, in brain tumor segmentations, the volume of the tumor is normally a small fraction of the grid size and the rest is background. The background should not be included in the computation.

Density and sparsity:

An algorithm could perform better with sparse point sets like geographical locations and worse with dense point sets like MRI segmentations and vice versa.

Generality:

Algorithms restricted to a special class of point sets cannot be applied in a general situation.

III. PROPOSED SYSTEM

Efficient Algorithm:

In day-to-day life, various image processing applications require distance over color codes. One of the distance is HDD. It can be considered as distance position of color codes present in terms of two point sets in the image. The HD can naturally be extended to the problem of finding the best partial distance from the given color code. A Hausdorff space is sometime called a separated space. The neighbourhood separate two points. A commonly used dissimilarity measure for comparing point sets and image segmentation is HD distance. A situation might arrive when very large point sets are compared using the Hausdorff distance but the computational complexity of H algorithm becomes an important issue. In this paper we proposed an Efficient algo for computing exact HD for rigid and non-rigid object. In the run time analysis a proposed algorithm has linear complexity for both rigid and non-rigid object. The proposed algorithm is tested for medical diagnosis application at "Image processing department".

Mathematical formulation:

```
for(i=0;i<n;i++)
{
    for(j=0;j<n;j++)
    {
        if(color found)
            calculate color position
            a. save first position as start point
            b. save last position as end point
            distance=a-b
    }
}
```

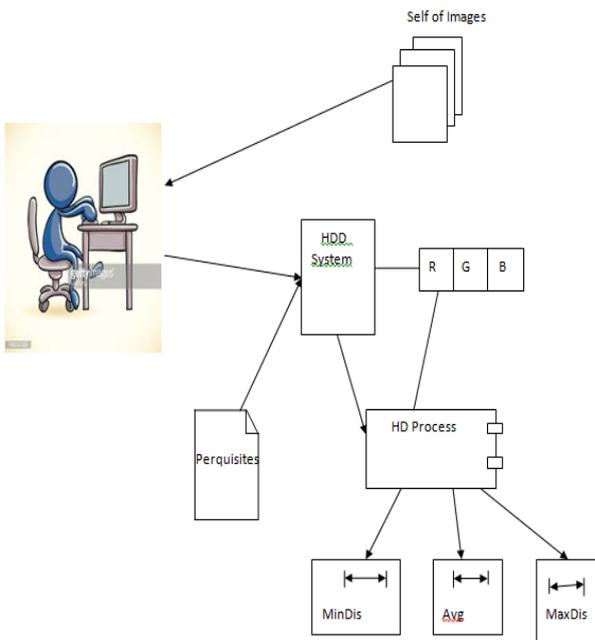


Fig 1: System architecture

IV. PROPOSED ALGORITHM

We propose a novel algorithm for calculating the exact Hausdorff distance.

Early Breaking

It is not always necessary that the scan in the inner loop runs completely through. Since the Hausdorff distance aims to find the maximum of the minimums, the inner loop can actually break as soon as a distance is found.

Random sampling in place of scanning

In random sampling, the aim is to avoid similar distances in successive iterations. This is achieved by randomly iterating the points in the inner loop. However, we randomize the sampling order also in the outer loop.

Excluding intersection

This optimization generally provides a small increase in speed beyond the combination of early breaking and randomization. It is not a core part of the general HD algorithm, and can be used to achieve a small speed increase in cases where it is applicable.

Runtime analysis

It is expected that the average case runtime is biased towards the best case because the worst case generally requires conditions that are more difficult to satisfy.

These above technique used for to implementing and calculating the correct Hausdorff distance.

NAIVEHDD straight forwardly computes the directed Hausdorff distance.

Steps:

Initially take two finite point sets A, B.

All points directed to the Hausdorff distance.

1. cmax as 0
2. for every point x of A do
3. cmin as infinity
4. for every point y of B do
5. value of d as a mod of(x,y)
6. if value of d is less than cmin then
7. cmin as d
8. end if
9. end for
10. if cmin is greater than cmax then
11. cmax as cmin
12. end if
13. end for

V. OBJECTIVE AND APPLICATION

Objective:

- Our main goal is to calculate the exact Hausdorff Distance between two point sets.
- To calculate the area or perimeter of any shape(polygon).

Application:

- Computed Tomography (CT scan)
- Image Matching
- Comparing images using hausdorff distance
- Robust face detection using hausdorff distance

VI. CONCLUSION

We propose an efficient algorithm for computing the exact Hausdorff distance. We formally show that the proposed algorithm has a nearly-linear runtime in the average case. The proposed algorithm combines early breaking and randomization optimizations to achieve a significant increase in speed over other algorithms that do not use this combination. The proposed algorithm does not impose any restrictions on the input data, and is hence generalizable to all applications. Moreover, it does not require a complex setup phase needing high computational effort and extensive storage space.

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